

Dynamic Mott gap from holographic fermions in geometries with hyperscaling violation

ZhongYing Fan

Department of Physics, Beijing Normal University, 100875 Beijing, China
zhyingfan@gmail.com

ABSTRACT: We investigate a dynamically generated Mott gap from holographic fermions in asymptotically geometries with hyperscaling violation by employing a bulk dipole coupling for the fermion field. We find that when the coupling strength increases the spectral function first transfers to the negative frequency region but soon redistributes to the positive region. A stable gap and two bands emerges for all momentum when the coupling strength beyonds a critical value. Generally, The upper band on the positive frequency axis is much sharper than the lower band on the negative side. When the diploe coupling increases further, the gap becomes larger and the up band still keeps sharp while the lower band disperses and widens, concentrating on the small momentum region. We also find that the bands will be smoothed out gradually with the increasing of hyperscaling violation.

KEYWORDS: AdS/CFT correspondence, gauge/gavity duality, holography and condensed-matter theory.

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1. Introduction

Recent years, AdS/CFT correspondence has been widely used to study condensed-matter theory (AdS/CMT). The strongly coupled conformal theory in the boundary is mapped to weakly coupled gravity theory in the bulk. With this great advantage, people have successfully constructed holographic models of Fermi and non-Fermi liquids in kinds of geometries[1, 2, 3, 4, 5, 6, 7, 8, 9] and can analytically investigate the liquids properties, showing the dispersion relation and the width of the quasi-particle like excitation.

Since condensed-matter systems are usually described by non-relativistic field theories, to search more proper gravity duals people have further generalized the correspondence to non-relativistic holography, typically with anisotropic scaling behaviors for temporal and spatial coordinates[10, 11, 12, 13] i.e. Lifshitz-like geometry with dynamical exponent. For realistic systems, importantly, another exponent, called hyperscaling violation will emerge and play a crucial role in low energy physics. It is certainly necessary to extend holography to this non-trivial case and it indeed comes true by employing the standard Einstein-Maxwell-dilation action in the bulk[4, 14, 15, 16, 17]. The metric behaves like:

$$ds^2 = -\frac{dt^2}{r^{2m}} + r^{2n}dr^2 + \frac{dx_i^2}{r^2} \quad (1.1)$$

where $i = 1, 2, \dots, d$ is space index, m and n are related to dynamical exponent z and hyperscaling violation exponent θ by

$$z = \frac{m + n + 1}{n + 2}, \quad \theta = \frac{n + 1}{n + 2} \cdot d \quad (1.2)$$

Note when $n = -1$, the metric reduces to the pure Lifshitz spacetime and $n = -2$ corresponds to a class of spacetime conformally related to $AdS_2 \times R_d$ with the locally critical limit $z \rightarrow \infty$, $\theta \rightarrow -\infty$, while z/θ fixed to be a constant[18]. The metric transforms as

$$t \rightarrow \lambda^z t, \quad x_i \rightarrow \lambda x_i, \quad r \rightarrow \lambda^{(d-\theta)/d} r, \quad ds \rightarrow \lambda^{\theta/d} ds \quad (1.3)$$

Clearly, the metric is not scale invariant, unlike the pure Lifshitz case. The dual boundary theory exhibits this peculiar behavior generally just below some non-trivial dimensional scale. But we won't consider this complication, simply assuming the metric is asymptotically geometries with hyperscaling violation for our purpose.

We have found remarkable influence of hyperscaling violation on the dynamical gap by introducing a magnetic dipole coupling for bulk fermions which was first proposed in [19, 20] and further studied in [21, 22, 23, 24]. Similar to the published work, a gap in the spectral function was opened when the dipole coupling strength p exceeds some critical value, very like a Mott insulator and further widened with p increasing. The coupling strength p plays a role similar to the dimensionless interaction strength U/t in the Hubbard model of fermions. The novel feature we find is that two bands exist in the spectral function, a upper band on the positive frequency axis and a lower band on the negative side, respectively, and behave qualitatively different with the increasing of the interaction p and hyperscaling violation θ .

This paper is organized as follows: In section 2, we briefly review the effective gravity model i.e. Einstein-Maxwell-dilaton theory for geometries with hyperscaling violation. In section 3, we study the bulk fermions with a dipole interaction, deriving the equations of motion for the retarded correlator. In section 4, we numerically solve the equations of motion under proper boundary conditions and extract the main results of the emergence of the gap. Finally, we present a conclusion in section 5.

2. Effective Gravity Model

The standard Einstein-Maxwell-dilaton (EMD) action reads

$$S = \int d^{d+2}x \sqrt{-g} [R - 2(\partial\phi)^2 - V(\phi) - \frac{\kappa^2}{2} Z(\phi) F^2 - \frac{\kappa^2}{2} H^2] \quad (2.1)$$

where the AdS radius has been set to 1. The solutions with hyperscaling violation are listed in the following:

$$ds^2 = -r^{-2m}h(r)dt^2 + r^{2n}h^{-1}(r)dr^2 + \frac{dx_i^2}{r^2}, \quad h(r) = 1 - \left(\frac{r}{r_h}\right)^\delta \quad (2.2)$$

$$F^{rt} = F_0 r^{(m-n+d)} Z^{-1}(\phi), \quad H^{rt} = H_0 r^{(m-n+d)} \quad (2.3)$$

$$\phi = k_0 \log r, \quad k_0 = \sqrt{\frac{d}{2}(m-n-2)} \quad (2.4)$$

$$V(\phi) = -V_0 e^{-\beta\phi}, \quad V_0 = \delta(m+d-1), \quad \beta = \frac{2(n+1)}{k_0} \quad (2.5)$$

$$Z^{-1}(\phi) = Z_0 e^{-\alpha\phi} + Z_1, \quad \alpha = \frac{2(n+d+1)}{k_0}, \quad Z_0 = \frac{\delta(m-1)}{\kappa^2 F_0^2}, \quad Z_1 = -\frac{H_0^2}{F_0^2} \quad (2.6)$$

where $\delta = m+n+d+1$, r_h is the location of the horizon, F_0, H_0 are constants which are proportional to the conserved charges carried by the black brane. The Hawking temperature and the entropy density of the black brane are give by

$$T = \frac{\delta}{4\pi} \frac{1}{r_h^{(m+n+1)}}, \quad s = \frac{1}{8\kappa^2 r_h^d} \quad (2.7)$$

In the zero temperature limit ($r_h \rightarrow \infty$), the entropy density approaches to zero, qualitatively differing from the black holes in RN background and more important for realistic systems with degenerate ground states.

In order to admit a stable theory, the dilaton solution is required to be real, leading to $m \geq n+2$ or equivalently $z \geq 1 + \theta/d$, $\theta < d$. Moreover, in the boundary limit $r \rightarrow 0$, the field strength $F^{\mu\nu}$ diverges such that the dual chemical potential cannot be well defined. Therefore we introduce another gauge field $H = dB$ to obtain a proper definition for finite density

$$B(r) = \mu \left(1 - \frac{r^{(d-m+n+1)}}{r_h^{(d-m+n+1)}}\right) dt \quad (2.8)$$

where μ is the chemical potential. The constrained condition satisfying B and H finite in the UV limit is

$$2 \leq m-n \leq d, \quad d \geq 3 \quad (2.9)$$

The divergent behavior of the field $F^{\mu\nu}$ certainly needs to be treated properly in a holographic renormalization procedure which we won't discuss in detail in this paper. Since the bulk fermions we consider don't couple to the dilaton and F fields directly, the results we obtain are still credible, in the absence of a full treatment of the holographic EMD theory.

3. Holographic fermion with magnetic dipole coupling

In order to explore the effects of magnetic dipole coupling on the spectral function of fermions, we start from the following action

$$S_f[\Psi] = i \int d^{d+2}x \sqrt{-g} \bar{\Psi}(\Gamma^a \mathcal{D}_a - M - ip\mathcal{H})\Psi + S_{bdy}[\Psi] \quad (3.1)$$

$$S_{bdy}[\Psi] = i \int_{\epsilon} d^{d+1}x \sqrt{-g_{\epsilon}} \sqrt{g^{rr}} \bar{\Psi}_+ \Psi_- \quad (3.2)$$

where S_{bdy} is a boundary action to ensure a well defined variational principle[25] for the total fermion action. And $\bar{\Psi} = \Psi \Gamma^t$, $\mathcal{D}_a = (e_a)^\mu D_\mu$, with $D_\mu = \partial_\mu - iqB_\mu + \frac{1}{4}\omega_{\mu ab}\Gamma^{ab}$, $\Gamma^{ab} = \frac{1}{2}[\Gamma^a, \Gamma^b]$. $\omega_{\mu ab}$ is the spin connection 1-form and $\mathcal{H} = \frac{1}{2}\Gamma^{ab}(e_a)^\mu(e_b)^\nu H_{\mu\nu}$. Γ^a are the $d+2$ dimensional gamma matrices and $(e_a)^\mu$ are the vielbeins and M is the mass of the probe fermion. Furthermore, g_ϵ is the determinant of the induced metric on the constant r slice, $r = \epsilon$. Ψ_\pm is defined by

$$\Psi_\pm = \frac{1}{2}(1 \pm \Gamma^r)\Psi, \quad \Gamma^r \Psi_\pm = \pm \Psi_\pm \quad (3.3)$$

The Dirac equation derived from the action reads

$$(\Gamma^a \mathcal{D}_a - M - ip\mathcal{H})\Psi = 0 \quad (3.4)$$

Taking a Fourier transformation

$$\Psi(r, x_\mu) = (-gg^{rr})^{-\frac{1}{4}} e^{-i\omega t + ik_i x^i} \psi(r, k_\mu), \quad k_\mu = (-\omega, \vec{k}) \quad (3.5)$$

where the prefactor was introduced to remove the spin connection in the equations of motion. Since the theory is rotational invariant, we can choose the momentum along the x_1 direction. The Gamma matrices are chosen as follows

$$\Gamma^r = \begin{pmatrix} -\sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}, \quad \Gamma^t = \begin{pmatrix} i\sigma^1 & 0 \\ 0 & i\sigma^1 \end{pmatrix}, \quad \Gamma^{x_1} = \begin{pmatrix} -\sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix} \quad (3.6)$$

where σ are Pauli matrices. We further set

$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}, \quad \psi_{\pm} = \begin{pmatrix} u_{\pm} \\ d_{\pm} \end{pmatrix} \quad (3.7)$$

Since the Dirac equation is first order, there exists some relation between ψ_+ and ψ_- . Assuming $\psi_+(r, k_{\mu}) = -i\xi(r, k_{\mu})\psi_-(r, k_{\mu})$, we can derive an elegant equation to extract correlators

$$\sqrt{g^{rr}}\partial_r\xi_{\pm} + 2M\xi_{\pm} = (v_- \pm k\sqrt{g^{x_1x_1}})\xi_{\pm}^2 + (v_+ \mp k\sqrt{g^{x_1x_1}}) \quad (3.8)$$

where $\xi_+ = iu_-/u_+$, $\xi_- = id_-/d_+$, ξ_{\pm} are the eigenvalues of the matrices ξ . And

$$v_{\pm} = \sqrt{-g^{tt}}(\omega + qB_t \pm p\sqrt{g^{rr}}\partial_r B_t) \quad (3.9)$$

The corresponding retarded functions can be readily obtained as follows[26]

$$G_O(k_{\mu}) = \lim_{r \rightarrow 0} \xi(r, k_{\mu}) \quad (3.10)$$

At the event horizon, we impose in-falling boundary conditions

$$\xi(r_h, k_{\mu}) = i, \quad \text{for } \omega \neq 0 \quad (3.11)$$

We emphasize that the dimension of the fermionic operator O is $\Delta = (m + d)/2$ which leads to the unitarity bound was automatically satisfied with $m \geq 0$ given by the null energy conditions[26]. The fermion mass decouples from the operator UV dimension, contributing only to the IR physics which is peculiar in the asymptotical geometries with hyperscaling violation.

4. Numerical results and Emergence of the gap

To extract the effects of bulk dipole coupling on the spectral function, we need to numerically solve the flow equation (3.8) with initial conditions (3.11). The spectral function is proportional to $ImG(\omega, k)$, up to normalization. Due to the relation $G_{11}(\omega, k) = G_{22}(\omega, -k)$, we will only consider $G_{22}(\omega, k)$ and omit the subscript in the following. For convenience to perform numerical calculation, we set $M = 0, \mu = 1, q = 2, z = 2, d = 3$. The dipole interaction strength p and hyperscaling violation θ (or n) remain to be free.

First, we fix hyperscaling violation, considering $n = 0$ case. From the plots above in figure 1, we find a sharp quasi-particle like peak at $k_F \approx 1.2044$ for $p = 0$, indicating a Fermi surface. The dual liquid is of Fermi type with linear dispersion relation at the maximum height of the spectral function. For larger charge q , it has been investigated

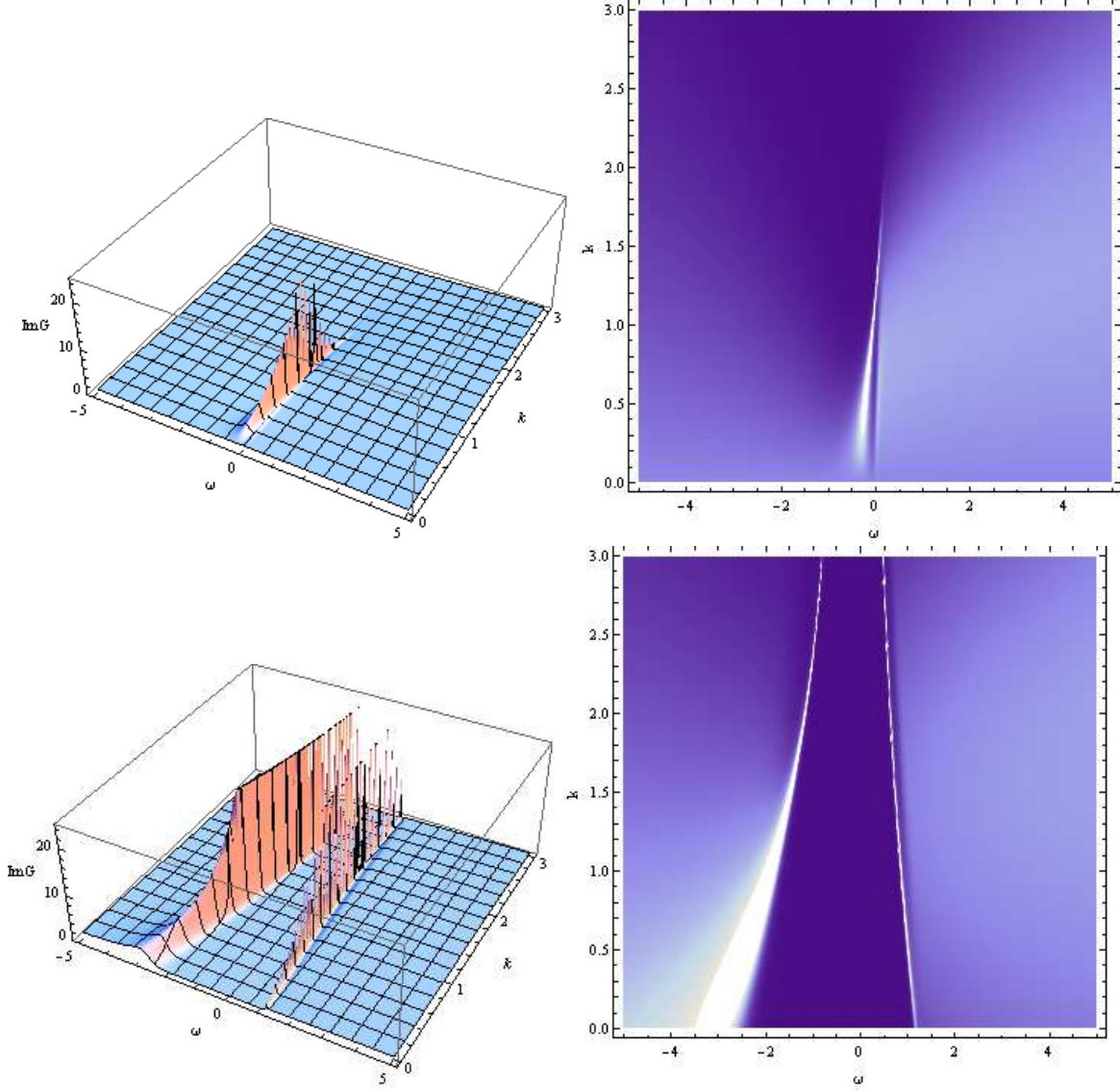


Figure 1: The 3D and density plots of $\text{Im}G(\omega, k)$. In the plots above, $p = 0$, a sharp peak occurs at $k \approx 1.2044$. In the plots below, $p = 4$, a gap emerges around $\omega = 0$.

with great detail in [26] that more branches of Fermi surfaces will emerge and even a peculiar Fermi shell-like structure, which exactly contains many sharp and singular peaks in some narrow interval of the momentum space exists when q is large enough.

In the plots below of figure 1, a gap emerges when the interaction strength was turned on $p = 4$ and there are two bands, located at positive frequency (we call it upper band) and negative frequency (called lower band) axis respectively. Interestingly, the upper band is much sharper than the lower one. For bigger momentum, the lower band

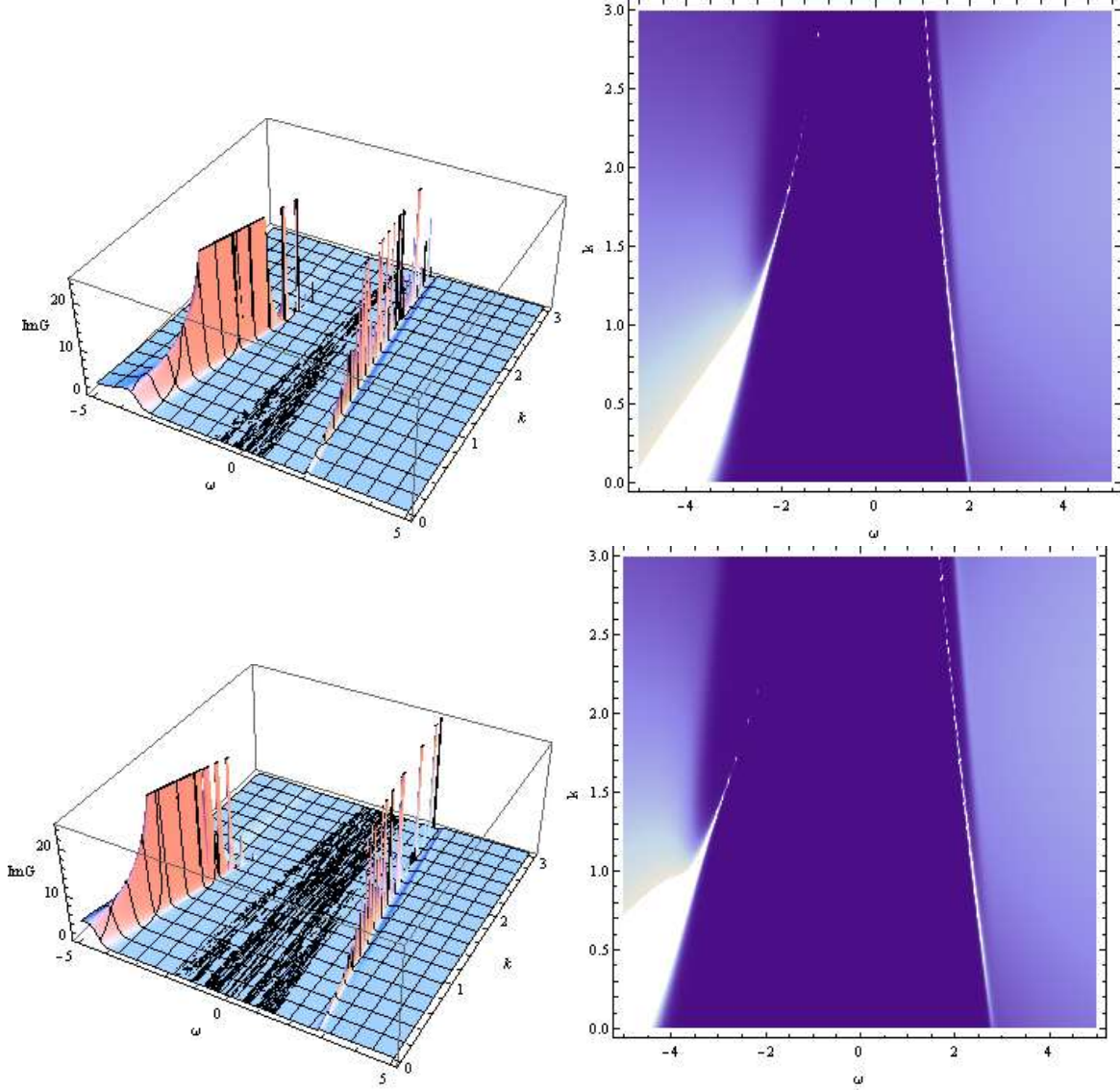


Figure 2: The 3D plots and density plots of $ImG(\omega, k)$ for $p = 6$ (plots above) and $p = 8$ (plots below).

is also as sharp as the upper band but becomes widened for relative small momentum, probably implying that the intensity of the spectral function tends to distribute at small momentum region in the negative frequency region.

From plots in figure 2¹, we can see that when p increases further, the gap becomes larger. The upper band always keeps sharp for all momentum, just translationally moving to the higher frequency region while the lower band is deformed much evidently

¹The black part in the 3D plots are purely numerical noise.

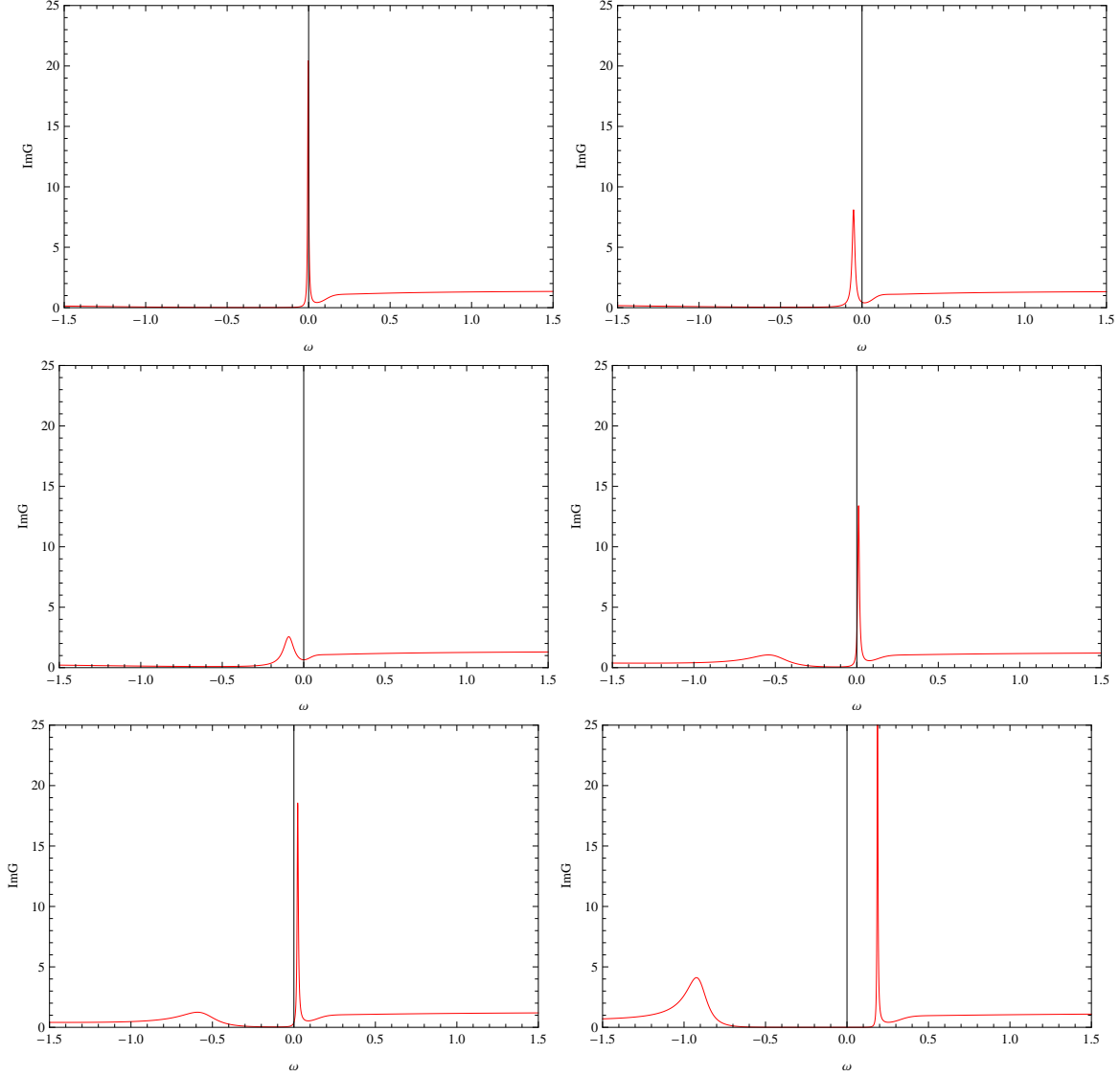


Figure 3: The plots of $ImG(\omega, k)$ for $p = 0.01, 0.2, 0.4, 1.1, 1.2, 2.0$, from left to right and from top to down, respectively.

by transfer of the spectral weight dominantly over the small momentum and negative high frequency region such that the intensity in the big momentum almost disappears.

To explore further the distribution of the spectral function in detail when p increases, we show the plots of spectral function in figure 3. For very small p , the spectral function still has a sharp peak at $\omega = 0$, showing the main feature of a Fermi surface. As p increases, the intensity of the peak degrades and first transfers to negative frequency region but soon returns to the positive frequency region when p increases

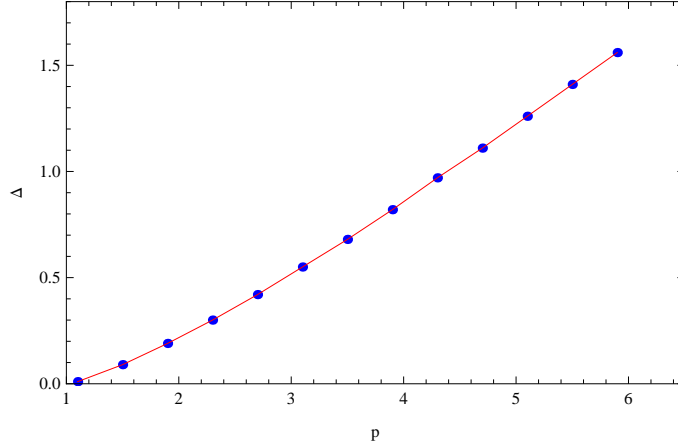


Figure 4: The gap width Δ as a function of p .

further. Finally, around $p_{crit} \approx 1.2$ the original sharp peak at $\omega = 0$ disappears and two stable bands structure emerges in both positive and negative frequency regions. We observe that the dominant spectral density appears at the upper band. Furthermore, we show in figure 4 that the width of the gap Δ becomes wider as interaction strength p increases.

We now turn to vary hyperscaling violation θ with fixed p . Without loss of generality, we set $p = 6$. In figure 5, we show the 3D plots and density plots of the spectral function for $n = 0.5$ in the plots above and $n = 1$ in the plots below respectively. The gap width keeps almost independent of the hyperscaling violation while the bands structure are gradually smoothed out as n increases. We clearly find that the sharper upper band disappears much easier than the lower band. The similar phenomenon also happens to the lower band itself. As n is amplified, the relative sharper region of the lower band in high momentum disperses and becomes wider while the original rough region in the small momentum space becomes narrower and narrower, implying the effects of smoothing. We may argue that in the $\theta \rightarrow d$ limit, any sharp peak of the spectral function will be completely smoothed out and the spectral density will transfer and redistribute to all frequency-momentum space homogeneously, with no explicit gap and bands structure again, probably indicating a critical phase.

5. Conclusions

In this paper, we have studied the novel features of fermions in the presence of bulk dipole coupling in the geometries with hyperscaling violation. For a finite hyperscaling violation $\theta = d/2$, we observe that when the dipole interaction strength $p=0$, a sharp

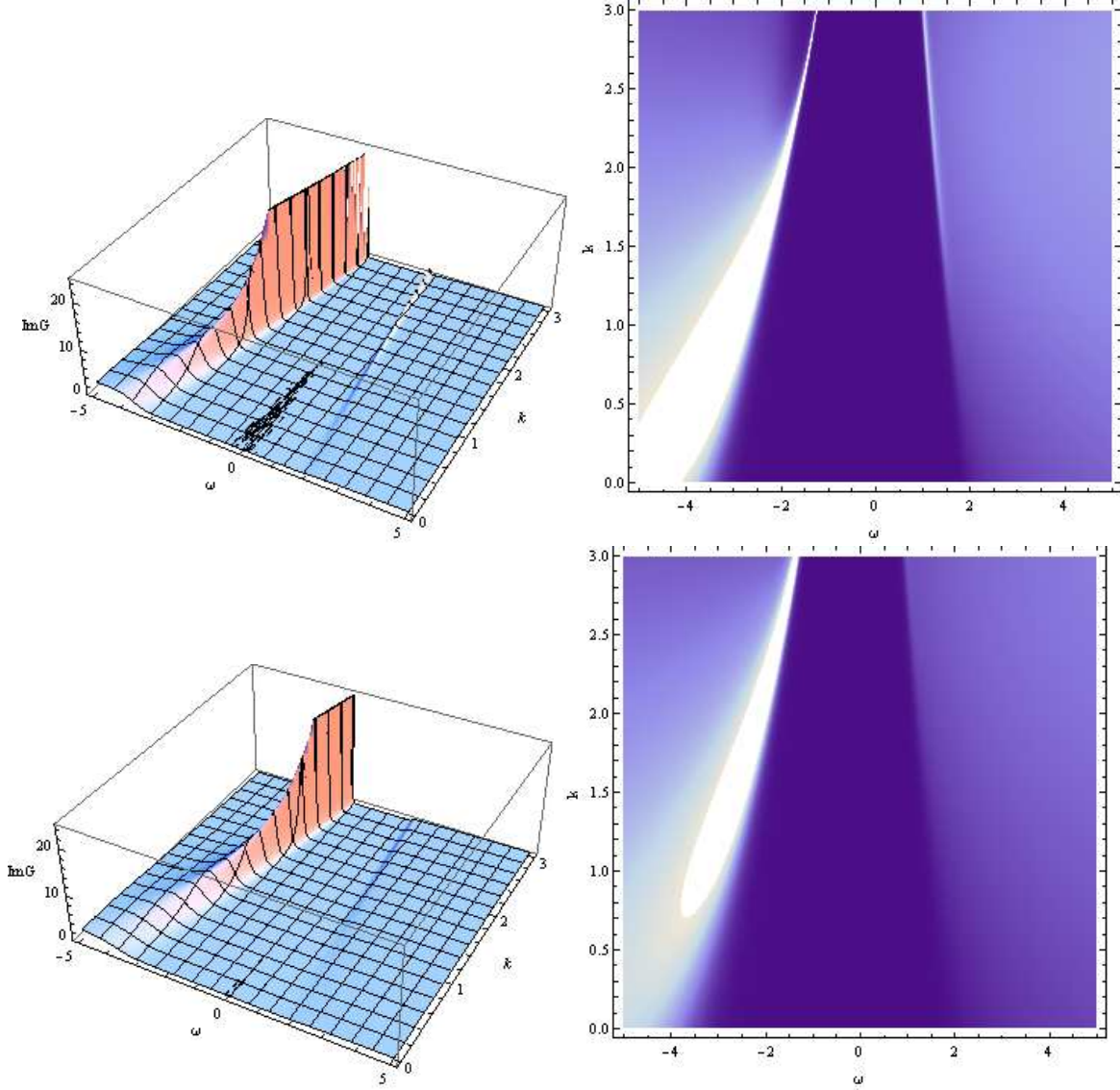


Figure 5: The 3D plots and density plots of $ImG(\omega, k)$ for $n = 0.5, 1$ in the plots above and below, respectively.

quasi-particle like peak occurs near $k_F \approx 1.2044$ at zero frequency, indicating a Fermi surface. As p increases, the intensity of the sharp peak degrades and first transfers to negative frequency region but soon redistributes and returns to positive frequency space. When p crosses a critical value p_{crit} , the Fermi sea disappears and a stable gap with two bands located on both positive and negative frequency axis respectively emerges for all momentum. It seems very interesting that these two bands behaves quite differently. The upper band on the positive frequency region keeps sharp for all

momentum while the lower band on the negative frequency region is only sharp for high momentum but disperses in the small momentum space. As p increases further, the gap becomes wider and the upper band is still sharp but the lower band redistributes the intensity of the spectral function to smaller momentum space by sacrificing the sharp region of the high momentum. We also find that the width of the gap increases with the increasing of p .

When we fix $p = 6$ and turn on larger hyperscaling violation $n = 0.5, 1$ respectively, the gap almost doesn't change but the peaks and bands become smooth gradually. More interesting, the relative sharper upper band disappears first while the lower band also becomes narrower and narrower. Thus, the strength of the spectral density might distribute homogeneously to all frequency-momentum space in the $\theta \rightarrow d$ limit. It is of certain interests to explore the postulated critical phase in this limit. We leave it in near future.

6. Acknowledgments

I would like to thank Professor Sije Gao for his useful suggestions and encouragement. This work is supported by NSFC Grants NO.10975016, NO.11235003 and NCET-12-0054.

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